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***Abstract*— The analog electronic circuits are always subject to some imperfections. Analog imperfections cause deviations from nominal values of electronic elements. In the case of Linear Time-Invariant (LTI) circuits, the coefficients of the transfer function include some deviations from related typical values leading to the differences between the typical (i.e. design) and the actual transfer functions. In this paper, the analog imperfections are digitally estimated using only the output samples, without any access to the input signal nor to the analog system (blind method). Super Exponential Algorithm (SEA) is used as the blind equalization technique, since it provides rapid convergence. The only assumption is that the input is a non-Gaussian independent and identically distributed (i.i.d.) signal. Using this algorithm, the effects of analog imperfections in the analog circuits can be digitally estimated and possibly compensated without any dependance on the types and the sources of the analog imperfections. It provides the possibility to have an online compensation of the imperfections (realization errors, drifts, etc.). The analog imperfections have been estimated with a precision of $\pm 0.2\%$ and $\pm 1.3\%$ for the exemplary RC and RLC circuits respectively.**

I. Introduction

Analog electronic circuits are always subject to some random deviations from the nominal values of their components. Analog imperfections cause some unknown deviations from typical values of the coefficients related to the nominator and denominator of the transfer function associated with the LTI analog circuit. These unknown deviations are originated from various sources. The imperfections related to fabrication are generally independent of time. There are also some time-varying contributions in the deviations from typical values as well. Time-varying imprecisions like temperature drifts appear because of ambiance conditions. In practice, there are many applications in which high sensitivity to the parameters of the associated analog circuits is a bottleneck. As an example, the high sensitivity to analog imperfections is a major problem in sigma-delta A/D converters [1]. The proposed solutions for handling this problem are mostly based on the calibration techniques which are costly and not efficient [2]. Moreover, these methods are very dependent on the type and the source of the analog imperfections [3]. Hybrid Filters Banks (HFB) are exploited in the architecture of the wide band A/D converters [4] but HFB-based A/D converters have not been practically used because of intolerable amount of sensitivity to analog imperfections [4]. Calibration method has been proposed but is not a generic solution and is dependent on the source of the imperfection [4]. Accordingly, the necessity of a general digital solution for the estimation of analog imperfections is apparent. So, analog imperfections could be compensated through the estimated deviations. The objective of this work is to estimate the imperfections of the analog circuits and thus the respective actual transfer function using only the samples of the analog system output.The estimation algorithm has to be independent of the types and the sources of the analog imperfections. Second-Order Statistics (SOS) have been previously used to provide a nonlinear model for the analog circuits imperfections [5]. Since the variations of the signal power associated with the analog imperfections are so little and the proposed model is based on signal powers (variances), then that model is not totally satisfactory. In this paper, Higher-Order Statistics (HOS) parameters are used for the estimation of the analog imperfections through a blind equalization technique. The input signal is supposed to be non-Gaussian at the next sections as HOS (>2) parameters of Gaussian processes are null [6]. Blind equalization technique is the core of the proposed algorithm.

II. Estimation of analog imperfections and blind equalization

A. Problem definition

Considering figure 1, it is supposed that the Nyquist sampling rate has been respected. It is assumed that the sampled

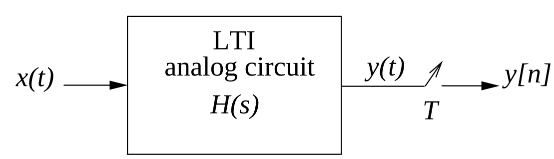

Fig. 1. An arbitrary LTI analog circuit with transfer function of $H(s)$. $y[n]$ represents the output after sampling.

version $y[n]$ of the output is the only available signal. Transfer function of the analog circuit $H(s)$ includes some unknown imperfections. It is supposed that there are totally K unknown coefficients in the nominator and the denominator of $H(s)$. Real value of i^{th} unknown coefficient α_i can be considered as:

$$\alpha_i = \alpha_{i_\circ}(1 + \delta_{\alpha_i}) \qquad 1 \le i \le K \tag{1}$$

where $\alpha_{i_\circ}$ stands for the typical (or nominal) value of this coefficient. δ_{α_i} is the relative imperfection (or relative deviation from typical value) associated with α_i. The problem is here to estimate the unknown relative imperfections $\{\delta_{\alpha_1}, \delta_{\alpha_2}, \cdots, \delta_{\alpha_K}\}$ using only the samples $y[n]$ of analog output. In practice, the nominal values $\{\alpha_{1_\circ}, \alpha_{2_\circ}, \cdots, \alpha_{K_\circ}\}$ are a priori known, although they are not necessarily required for the proposed algorithm. The structure of the analog circuit (or simply the order of the nominator and the denominator of $H(s)$) is known since it is defined at the design phase. The extraction of the unknown relative imperfections from the inverse FIR filter (result of blind equalization) is facilitated through this information (see section III).

B. Blind equalization for the estimation of imperfections

Blind deconvolution or equalization is referred to the case when the input of an unknown LTI system is required to be reconstructed using only the output signal. SOS-based methods are not useful unless the unknown LTI system is minimum-phase [6]. Thus, the equalization is mostly implemented using HOS-based techniques. Regarding to the properties of HOS, cumulants and polyspectra are blind to any Gaussian process because all cumulants of the order higher than two are equal to zero for a Gaussian process [6]. Accordingly, the input would be supposed a non-Gaussian i.i.d. process for implementing the blind equalization. Equalization problem is simply equal to finding the inverse filter of an unknown system. This inverse filter is often considered as an FIR filter called equalizer filter (fig. 2). To realize this procedure, a criterion or objective function is considered. Equalization is realized through optimizing the equalizer filter $f[n]$ so that the criterion function is maximized (or minimized for Constant Modulus Algorithm (CMA) criterion) [6]. Criteria are some specific functions in terms of the cumulants due to $y[n]$ and $\widehat{x}[n]$. The third order cumulants are null for the signals with symmetric distributions [6]. Therefore, fourth order cumulant of $\widehat{x}[n]$ is used in this paper as the analog input has been considered with a uniform distribution. Super-Exponential Algorithm (SEA) proposed by Shalvi and Weinstein has been used in order to have a rapid convergence [7]. This algorithm provides an iterative procedure for updating the coefficients of equalizer filter. Before implementation of the updating algorithm, it is required to calculate the vector of input/output cross cumulant (fourth-order cumulant) $\mathbf{d}$ and the matrix of output covariance $\mathbf{R}$. The current value of the equalizer filter $\mathbf{f} = [f_0, f_1, \cdots, f_{L-1}]^T$ is used in these calculations. L is the length of the equalizer filter $\mathbf{f}$. Using cumulant operation $cum(\cdot)$, the i^{th} element d_i of the vector $\mathbf{d}(L \times 1)$ is obtained as follows:

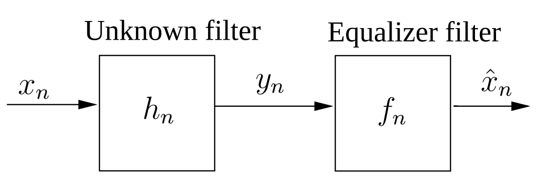

Fig. 2. Blind equalization system. Equalizer filter f_n is an FIR filter with length L. $\widehat{x_n}$ approximates the unknown input signal x_n in this system.

$$d_i = cum(\hat{x}_n, \hat{x}_n, \hat{x}_n, y_{n-i}) \quad 0 \le i \le L-1 \tag{2}$$

Each element R_{ij} of the covariance matrix $\mathbf{R}$ $(L \times L)$ is calculated as following:

$$R_{ij} = \frac{cum(y_{n-i}, y_{n-j})}{\sigma_x^2} \tag{3}$$

where σ_x^2 stands for the variance of unknown input. If σ_x^2 is not a priori known, it can be substituted with any positive real number in the equation 3. In this case, there would exist an ambiguity on the amplitude. In other words, the estimated inverse filter $f[n]$ would be an amplitude-scaled version of the exact inverse filter. Now, the iterative algorithm of SEA for finding the updated value of equalizer $\mathbf{f}_{new}$ is implemented as follows [7]:

$$\mathbf{V} = \mathbf{R}^{-1}\mathbf{d} \tag{4}$$

$$\mathbf{f}_{new} = \frac{1}{\sqrt{\mathbf{V}^{+}\mathbf{R}\mathbf{V}}}\mathbf{V}$$

where $(\cdot)^{+}$ denotes for transpose-conjugate operation and $\mathbf{V}$ $(L \times 1)$ is an intermediate vector. It should be noted that the old value of equalizer vector is implicitly incorporated in equation 4 through taking part in the calculation of $\mathbf{d}$ and $\mathbf{R}$. There is only one converging point which is associated with the inverse filter [6]. However, this algorithm may in practice converge to false results (spurious local maxima) for reasons such as inappropriate length of equalizer L, insufficient number of data utilized in the cumulant calculation, nonlinearities of the system and thus some initializations of the equalizer [6]. Initialization problem can be handled in the estimation of analog imperfections because the nominal analog system a priori is known. Hence, respective typical equalizer have been used as the initial equalizer.

III. IMPLEMENTATION OF THE ESTIMATION PROCEDURE

A. Estimation algorithm

Figure 3 shows the implementation. Equalizer filter $F(z)$ is supposed to be an FIR filter with length L. For estimating the imperfections of analog circuit, the procedure is realized in two phases. Firstly, blind equalization method (SEA procedure) is applied to the system as explained in the preceding section. It provides an FIR filter $f[n]$ which approximates the inverse filter associated with the analog circuit. At the second phase, the real coefficients of $H(s)$ are estimated. To better explain this stage, $G(s)$ is considered as the inverse function

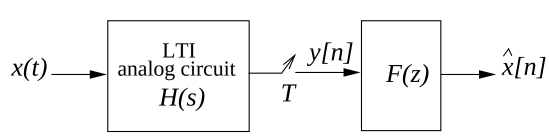

Fig. 3. An LTI analog circuit with transfer function of $H(s)$ to which the equalizer $F(z)$ has been applied.

of $H(s)$ with unknown coefficients. The nominator and the denominator orders of $G(s)$ are equal to the denominator and the nominator orders of $H(s)$ respectively. The coefficients of $G(s)$ are found through minimizing the error expression which follows:

$$G_{opt}(s) = arg\ min\ \ \|G(s) - F(e^{j\omega})\|_{s=j\frac{\omega}{T}} \quad w \in \rho \tag{5}$$

where T is the sampling period utilized in the first phase and ρ is the frequency band of interest. Depending on the transfer function of the analog system, ρ is appropriately selected so that the contribution of the unknown parameter is emphasized. For example, it can be concentrated about the nominal resonance frequency for an RLC circuit. The real coefficients of $H(s)$ and evidently the respective deviations from nominal values are obtained from $G_{opt}(s)$.

B. Simulation results

The algorithm that was explained in the previous section is now applied to several first- and second-order analog circuits. Firstly, a first-order RC circuit is considered. There are two parameters describing the transfer function of a general RC circuit: DC-gain g (gain at the zero frequency) and cut-off frequency ω_c. Respective transfer function can be described as follows:

$$H(s) = \frac{g\omega_c}{s + \omega_c} \tag{6}$$

For estimating the parameter of DC-gain (scale factor), it is required to know a priori the variance of the analog input (refer to section II-B). The first stage (blind equalization) was realized 1000 times for each deviation from nominal values using an FIR equalizer ($L = 9$). The algorithm converged to spurious local maxima (false results) in 5% of the times. Using an initial equalizer associated with nominal RC circuit (no deviation from nominal values) at the initialization procedure of blind equalization, the algorithm always converged to the global maximum. Figure 4 shows the histogram of the results for the realization of the algorithm supposing an RC circuit having 20% and 10% deviations from nominal cut-off frequency and DC-gain respectively. This histogram is in terms of the ratio of the estimated to real deviation from the nominal value. The histogram illustrates the distribution of the results due to 1000 sample paths of the noise. The average values of the results estimate the unknown deviations from nominal values (for DC-gain and cut-off frequency) with an error of -0.5% and 0.2% respectively. This simulation was implemented for different deviations from nominal values as well. The average estimation errors are shown in figure 5. The mean estimation error is always lower than 0.25% for 1000 sample paths of noise. Using larger repetition number in the simulation, the mean values will better approximate the deviation from nominal values. The algorithm was implemented for an RLC circuit as well (refer to figure 6). Related transfer function is described as following:

$$H(s) = \frac{\frac{\omega_r}{Q}s}{s^2 + \frac{\omega_r}{Q}s + \omega_r^2} \tag{7}$$

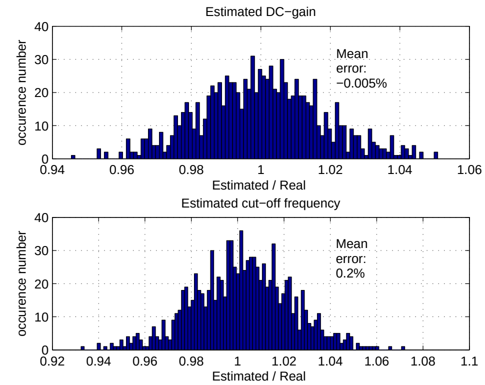

Fig. 4. Histogram due to the ratio of estimated to real deviation from nominal values after 1000 sample paths of noise for an RC circuit. The real deviation from the nominal values are 20% and 10% for the cut-off frequency and the DC-gain respectively.

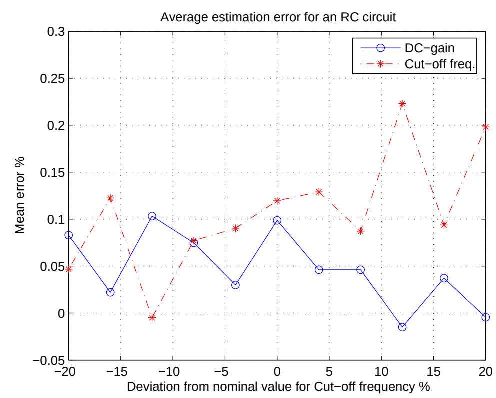

Fig. 5. Average errors of the estimation due to the DC-gain (solid) and the cut-off frequency (dashed) versus the real values of the deviation from nominal cut-off frequency for the general RC circuits.

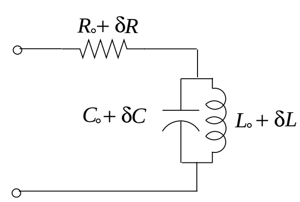

Fig. 6. The arbitrary RLC circuit used in the simulations. $\{R_\circ, L_\circ, C_\circ\}$ are the design values to which the unknown realization errors $\{\delta R, \delta L, \delta C\}$ are applied.

Deviations from nominal values for quality factor (Q) and resonance frequency (ω_r) were supposed as the unknown parameters. There is no need for the variance of input at the algorithm because the unknown parameters are independent of any scaling factor. Using a random initialization, the algorithm of blind equalization (first phase) converged to the spurious local maxima in 35% of times. Using nominal equalizer (related to the circuit with no deviations from nominal values), the rate of convergence to spurious local maxima changed. The percentage of convergence to the global maximum in terms of deviations from nominal frequency of resonance is shown in figure 7. However, converging to spurious local maxima causes no problem in practice even with random initialization because the incorrect equalizers are conveniently detected and put aside. Figure 8 illustrates the histogram of the results

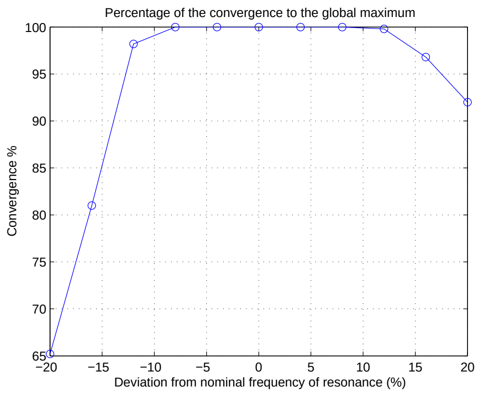

Fig. 7. The percentage of convergence to the global maximum. Horizontal axis shows the percentage of deviation from nominal frequency of resonance. Deviation from nominal quality factor is fixed (10%) and the algorithm is initialized by nominal values.

when deviations from nominal frequency of resonance and quality factor are supposed 20% and 10% respectively. The algorithm is repeated 500 times using an equalizer length of $L = 41$. The average error of estimation are 0.01% and -1.3% for frequency of resonance and quality factor respectively. Figure 9 shows the mean errors due to several implementation

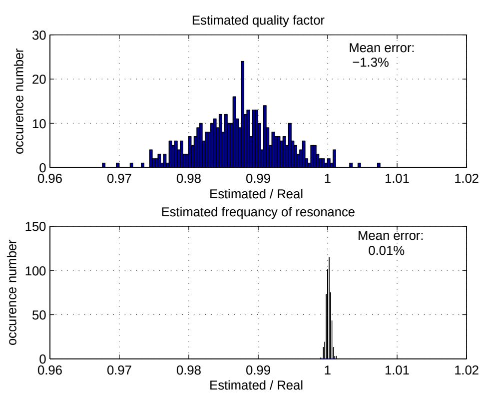

Fig. 8. Histogram due to the ratio of estimated to real deviation from nominal values after 500 sample paths of noise for an RLC circuit. The real deviation from the nominal values are 20% and 10% for the resonance frequency and the quality factor respectively.

of the algorithm supposing different values of deviations from nominal values. It is discerned that this algorithm (first phase) is very sensitive to the sampling period. In fact, the higher is the sampling frequency, the longer equalizer is required for compensating the lower levels of the spectrum amplitude at the frequency extremes (the frequencies near to $\pm\frac{\pi}{T}$). This is approved through analysis of the distribution of the mean errors particularly in figure 9. In the RLC case, the presence

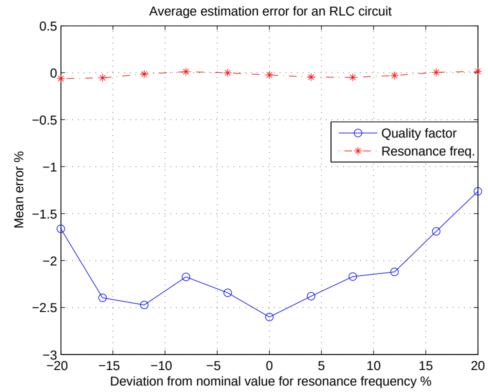

Fig. 9. Average errors of the estimation due to the quality factor (solid) and the resonance frequency (dashed) versus the real values of the deviation from nominal resonance frequency for the RLC circuits.

of a zero situated on the null frequency ($\omega = 0$) augmented the rate of convergence to spurious local maxima since the algorithm tries to compensate this zero (infinite gain for equalizer at zero frequency). In practice, the series resistance of the real inductance removes the associated zero in the spectrum.

IV. Conclusion

A method for estimating the imperfections of analog circuits (or their actual transfer function as well) was proposed in this paper using only the samples of the output and without any access to the input signal nor to the system components. This estimation method was proposed using blind equalization techniques. SEA algorithm for blind equalization was exploited to have a fast convergence. Several implementations of this method were realized using first- and second-order circuits. The analog imperfections of RC circuits were estimated with a mean error of -0.5% and 0.2% for DC-gain and cut-off frequency respectively. In the RLC case, the mean errors of estimation were 0.01% and -1.3% for frequency of resonance and quality factor respectively. This demonstrates the feasibility of digital compensation of analog imperfections.

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